Rational Numbers

Rational Number: a number that is the quotient of two integers

Operations with Rational Numbers

- Adding/Subtracting Rational Numbers:
  1. Use integer rules to determine whether to add or subtract
  2. Find a common denominator.
  3. Add or subtract numerators and keep denominator the same.
  4. Simplify if necessary.
  5. Use integer rules to determine the sign of the answer.

ex: $-2\frac{2}{5} + 3\frac{9}{10}$

Negative + Positive → Subtract

$$3\frac{9}{10} \rightarrow 3\frac{9}{10}$$
$$- \quad 2\frac{2}{5} = 2\frac{4}{10}$$
$$1\frac{5}{10} = \boxed{1\frac{1}{2}}$$

- Multiplying Rational Numbers:
  1. Convert mixed numbers to improper fractions.
  2. Cross-simplify, if possible.
  3. Multiply numerators and multiply denominators.
  4. Simplify if necessary
  5. Use integer rules to determine the sign of the answer

ex: $-2\frac{1}{2} \cdot 1\frac{1}{5}$

Negative · Positive = Negative

$$-\frac{5}{2} \cdot \frac{6}{5} = -\frac{3}{1} = \boxed{-3}$$

- Dividing Rational Numbers:
  1. Convert mixed numbers to improper fractions.
  2. Flip second fraction to its reciprocal and change division sign to multiplication
  3. Multiply the fractions (as above)

ex: $-4\frac{1}{4} \div (-4)$

Negative ÷ Negative = Positive

$$-\frac{17}{4} \div \left(-\frac{4}{1}\right)$$
$$\rightarrow -\frac{17}{4} \cdot \left(-\frac{1}{4}\right) = \frac{17}{16} = \boxed{1\frac{1}{16}}$$

Square Roots

Perfect Square: a number that is the product of a rational number multiplied by itself

ex: 1, 4, 9, 16, 25,...

1^2, 2^2, 3^2, 4^2, 5^2

Finding the square root of a number is the opposite of squaring a number. Find the number that you can multiply by itself to equal the given number.

ex: Find the square root

$$\sqrt{\frac{9}{64}}$$

Find the square root of the numerator & denominator:

$9 = 3^2$, so $\sqrt{9} = 3$

$64 = 8^2$, so $\sqrt{64} = 8$

$$\boxed{= \frac{3}{8}}$$

Determine whether the number is rational or irrational.

1. -83	2. $\sqrt{5}$	3. π	4. $\frac{7}{3}$

Evaluate each numerical expression.

5. $-\frac{4}{5} + \left(-\frac{3}{10}\right)$	6. $2\frac{3}{4} - 5\frac{1}{2}$	7. $-\frac{11}{15} \cdot \left(-\frac{10}{33}\right)$	8. $\frac{6}{7} \div (-4)$
9. $6\frac{2}{3} + \left(-9\frac{2}{9}\right)$	10. $-4\frac{1}{5} \div \left(-2\frac{1}{7}\right)$	11. $8\frac{2}{3} \cdot \left(-1\frac{2}{13}\right)$	12. $-3\frac{7}{12} - 5\frac{1}{8}$

Find each square root.

13. $\sqrt{25}$	14. $-\sqrt{144}$	15. $\sqrt{225}$	16. $-\sqrt{36}$
17. $\sqrt{121}$	18. $\sqrt{100}$	19. $\sqrt{\frac{1}{4}}$	20. $-\sqrt{\frac{49}{81}}$

Evaluating Algebraic Expressions

1. Substitute the given values for the variables in the expression
2. Evaluate the expression using the order of operations
   - Parentheses/Brackets (inside to outside)
   - Exponents
   - Multiplication/Division (left to right)
   - Addition/Subtraction (left to right)

ex: evaluate

$9x^2 - 4(y + 3z)$

for $x = -3, y = 2, z = 5$

$9(-3)^2 - 4(2 + 3 \cdot 5)$

$9(-3)^2 - 4(2 + 15)$

$9(-3)^2 - 4 \cdot 17$

$9 \cdot 9 - 4 \cdot 17$

$81 - 4 \cdot 17$

$81 - 68 = \boxed{13}$

The Distributive Property

1. Multiply the number outside the parentheses by each term in the parentheses.
2. Keep the addition/subtraction sign between each term.

ex: $5(8x - 3)$

$5(8x - 3)$

$5(8x) - 5(3)$

$\boxed{40x - 15}$

Simplifying Algebraic Expressions

1. Clear any parentheses using the Distributive Property
2. Add or subtract like terms (use the sign in front of each term to determine whether to add or subtract)

ex: $2(3x - 4) - 12x + 9$

$2(3x - 4) - 12x + 9$

$6x - 8 - 12x + 9$

$\boxed{-6x + 1}$

Evaluate each expression for $a = 9$, $b = -3$, $c = -2$, $d = 7$. Show your work.

21. $a - cd$	22. $2b^3 + c^2$	23. $\frac{a + d - c}{b}$	24. $(a - b)^2 + d(a + c)$
25. $4c - (b - a)$	26. $\frac{a}{b} - 5a$	27. $2bc + d(12 - 5)$	28. $b + 0.5[8 - (2c + a)]$

Simplify each expression using the Distributive Property.

29. $5(2g - 8)$	30. $7(y + 3)$	31. $-3(4w - 3)$	32. $(6r + 3)2$

Simplify each expression, showing all work.

33. $8(x + 1) - 12x$	34. $6w - 7 + 12w - 3z$	35. $9n - 8 + 3(2n - 11)$	36. $3(7x + 4y) - 2(2x + y)$
37. $(15 + 8d)(-5) - 24d + d$	38. $9(b - 1) - c + 3b + c$	39. $20f - 4(5f + 4) + 16$	40. $8(h - 4) - h - (h + 7)$

Solving One-Step Equations

1. Cancel out the number on the same side of the equal sign as the variable using inverse operations (addition/subtraction; multiplication/division)
2. Be sure to do the same thing to both sides of the equation!

ex: $-18 = 6j$

$$\frac{-18}{6} = \frac{6j}{6}$$

$$-3 = j \rightarrow \boxed{j = -3}$$

Solving Two-Step Equations

1. Undo operations one at a time with inverse operations, using the order of operations in reverse (i.e. undo addition/subtraction before multiplication/division)
2. Be sure to always do the same thing to both sides of the equation!

ex: $\frac{a}{7} - 12 = -9$

$$\frac{a}{7} - 12 = -9$$
$$+12 \quad +12$$

$$7 \times \frac{a}{7} = 3 \times 7$$

$$\boxed{a = 21}$$

Solving Multi-Step Equations

1. Clear any parentheses using the Distributive Property
2. Combine like terms on each side of the equal sign
3. Get the variable terms on the same side of the equation by adding/subtracting a variable term to/from both sides of the equation to cancel it out on one side
4. The equation is now a two-step equation, so finish solving it as described above

ex: $5(2x - 1) = 3x + 4x - 1$

$$10x - 5 = 3x + 4x - 1$$

$$10x - 5 = 7x - 1$$
$$-7x \quad -7x$$

$$3x - 5 = -1$$
$$+5 \quad +5$$

$$\frac{3x}{3} = \frac{4}{3}$$

$$\boxed{x = \frac{4}{3}}$$

Solve each equation, showing all work.

41. $f - 64 = -23$	42. $-7 = 2d$	43. $\frac{b}{-12} = -6$	44. $13 = m + 21$
45. $5x - 3 = -28$	46. $\frac{w + 8}{-3} = -9$	47. $-8 + \frac{h}{4} = 13$	48. $22 = 6y + 7$
49. $8x - 4 = 3x + 1$	50. $-2(5d - 8) = 20$	51. $7r + 21 = 49r$	52. $-9g - 3 = -3(3g + 2)$
53. $5(3x - 2) = 5(4x + 1)$	54. $3d - 4 + d = 8d - (-12)$	55. $f - 6 = -2f + 3(f - 2)$	56. $-2(y - 1) = 4y - (y + 2)$

Writing Expressions & Equations

Key Words for Operations:

Addition	Subtraction
• Sum • More than • Increased by • Plus	• Difference • Less than • Decreased by • Minus

Multiplication	Division
• Product • Times • Multiplied By • Twice/Doubled (x2)	• Quotient • Divided By • Half (÷2)

Writing Equations:

- Translate the equation using key words for operations.
- "Is" translates to =.

ex: Translate to an algebraic expression:

b multiplied by the sum of a and c

$$= \boxed{b(a + c)}$$

ex: Translate to an equation & then solve:

14 is 4 more than the product of x and 2.

$$14 = 2x + 4$$
$$-4 \qquad -4$$
$$\frac{10}{2} = \frac{2x}{2}$$
$$5 = x \rightarrow \boxed{x = 5}$$

Solving & Graphing Inequalities

1. Solve the inequality as if it is an equation.

2. If you multiply or divide both sides of the inequality by a negative number, flip the inequality sign to the opposite sign.

3. Write your answer with the variable to the left of the inequality sign.

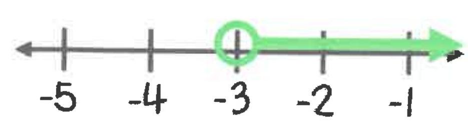
4. Graph the solution on a number line.
   - Make an open circle on the number if the number is not included in the solution (< or >)
   - Make a closed circle if the number is included (≤ or ≥)
   - Shade to the left for less than (< or ≤)
   - Shade to the right for greater than (> or ≥).

ex: $-5x - 3 < 12$

$$-5x - 3 < 12$$
$$+3 \quad +3$$
$$\frac{-5x}{-5} < \frac{15}{-5}$$

Divided both sides by a negative number, so flip the inequality sign!

$$\boxed{x > -3}$$



Translate each verbal phrase into an algebraic expression.

57. the difference of w and z	58. g more than the quotient of b and f	59. the product of v and n increased by p cubed

Translate each sentence into an algebraic equation. Then solve the equation.

60. Nine is six less than the quotient of x and negative two.	61. The sum of y and seven is twice the difference of y and twelve.

Solve each inequality. Then graph your solution on the number line.

62. $5x < -25$	63. $-18 + b \geq 23$
64. $j - 7 > -5$	65. $\frac{a}{-4} \geq 8$
66. $-2n - 24 \leq -36$	67. $18 < \frac{f}{3} + 20$

Scientific Notation

Standard Form to Scientific Notation: move the decimal after the first non-zero digit and eliminate any trailing zeros. Multiply by 10 to the power equal to the number of places you moved the decimal point. If the original number was greater than 1, the exponent is positive. If the number was less than 1, the exponent is negative.

ex: 0.0000571

0.00005.71

Original number < 1, so negative exponent

$= 5.71 \times 10^{-5}$

Scientific Notation to Standard Form: move the decimal point the number of places indicated by the exponent. If the exponent is positive, move the decimal right. If negative, move left.

ex: 3.5×10^3

Positive exponent, so move decimal right

$3.500. = 3{,}500$

Negative Exponents & Simplifying Monomials

Zero Exponent: Any number raised to the zero power equals 1

ex: $y^0 = 1$

Negative Exponent: Move the base to the opposite side of the fraction line and make the exponent positive

ex: $x^{-4} = \frac{1}{x^4}$

Monomial x Monomial: Multiply the coefficients and add the exponents of like bases

ex: $(4x^3)(2x^5) = 8x^8$

Monomial ÷ Monomial: Divide the coefficients and subtract the exponents of like bases

ex: $\frac{a}{a^6} = a^{-5} = \frac{1}{a^5}$

Power of a Monomial: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents

ex: $(-2fg^5)^3 = -8f^3g^{15}$

Power of a Quotient: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents

ex: $\left(\frac{5d^3}{c}\right)^2 = \frac{25d^6}{c^2}$

Convert each number to Scientific Notation.

68. 67,000,000,000	69. 0.0009213	70. 0.00000000004	71. 3,201,000,000,000,000

Convert each number to Standard Form.

72. 5.92×10^{-5}	73. 1.1×10^{7}	74. 6.733×10^{-8}	75. 3.27×10^{2}

Simplify each expression. Write your answers using only positive exponents.

76. w^{-9}	77. $\frac{m^5}{m^2}$	78. $f^5 \cdot f^3$	79. $\left(\frac{h^2}{g}\right)^3$
80. $(a^5)^2$	81. $\frac{1}{b^{-3}}$	82. z^0	83. $4r^6 \cdot 3r \cdot 2r^2$
84. $\frac{9p^{-2}}{3q^{-3}}$	85. $\frac{8d^3}{2cd^{-2}}$	86. $(g^4h)^2 \cdot (2g^3h^{-1})^2$	87. $(6a)^0$
88. $(-3n^2k^4)^2$	89. $\left(\frac{w^5 x^{-2} y}{w^2 x y^4}\right)^3$	90. $\frac{6 \cdot 10^7}{2 \cdot 10^3}$	91. $(1.5 \cdot 10^{-6}) \cdot (4 \cdot 10^9)$

Slope & Rate of Change

Finding the Slope Given Two Points: Use the coordinates from the points in the slope formula:

$$\text{Slope (m)} = \frac{y_2 - y_1}{x_2 - x_1}$$

ex: (4, -2), (-3, 8)
x_1 y_1 x_2 y_2

$$m = \frac{8-(-2)}{-3-4} = \frac{10}{-7} = \boxed{-\frac{10}{7}}$$

Finding the Rate of Change From a Table: Determine the amount the dependent variable (y) is changing and the amount the independent variable (x) is changing.

$$\text{Rate of Change} = \frac{\text{change in y}}{\text{change in x}}$$

ex:

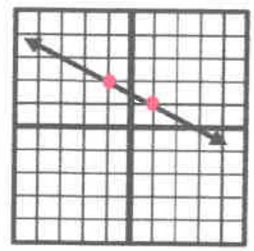
+2 +2 +2

x	# months	3	5	7	9
y	Cost ($)	80	130	180	230

+50 +50 +50

$$m = \frac{50}{2} = \boxed{\text{25 dollars/month}}$$

Finding the Slope From a Graph: Choose 2 points on the graph. Find the vertical change (rise) and horizontal change (run) between the 2 points and write it as a fraction $\frac{\text{rise}}{\text{run}}$. (Up is positive, down is negative, right is positive, and left is negative).



rise = +1
run = -2

$$m = \frac{1}{-2} = \boxed{-\frac{1}{2}}$$

Graphing Linear Equations

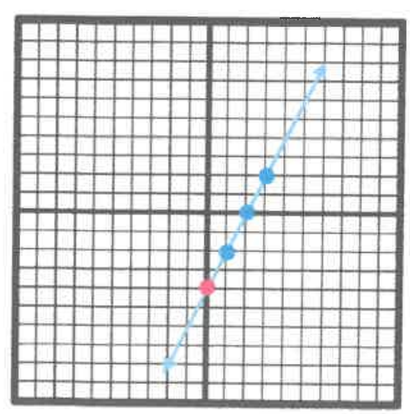
Slope-Intercept Form: $y = mx + b$

slope (m) y-intercept (b)

ex: $y = 2x - 4$

y-intercept: -4

slope: $2 = \frac{2}{1}$ ← rise ← run



How To Graph:

1. Make a point on the y-axis at the y-intercept.
2. Use the slope to determine where to make the next point. The numerator tells you the rise (how far up/down) and the denominator tells you the run (how far right/left) to make the next point.
3. Repeat to make more points and then connect the points with a line.

Find the slope of the line that passes through the points. Show your work.

92. (-5, 3), (2, 1)

93. (8, 4), (11, 6)

94. (9, 3), (9, -1)

95. (-4, -2), (-6, 4)

Find the rate of change. Show your work.

96.

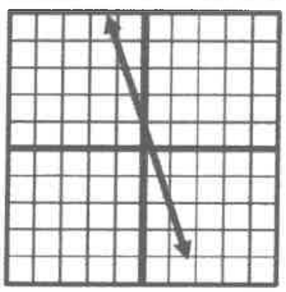
Number of Hours	3	6	9	12
Distance (in miles)	135	270	405	540

97.

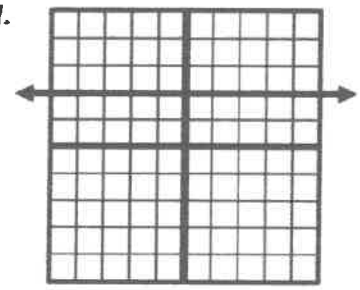
Number of Weeks	1	3	5	7
Pounds	173	169	165	161

Find the slope of the line.

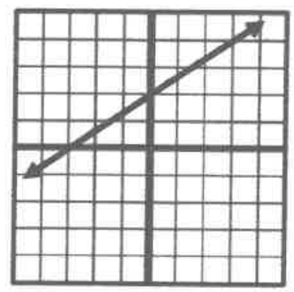
98.



99.

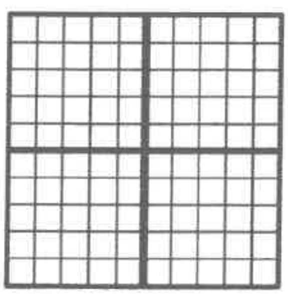


100.

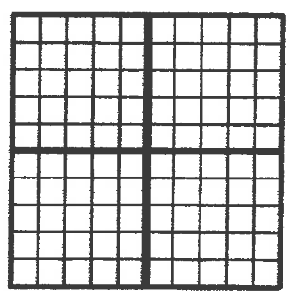


Graph the line.

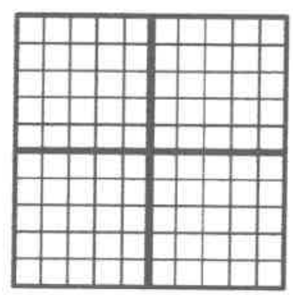
101. $y = -x - 3$



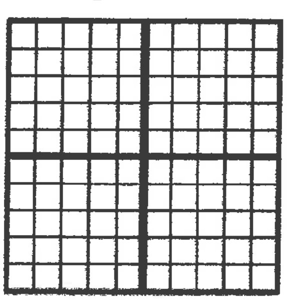
102. $y = \frac{1}{3}x + 2$



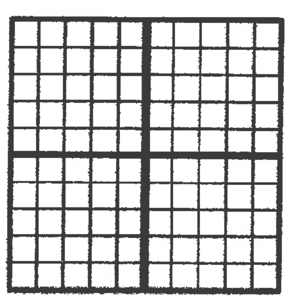
103. $y = -3x - 1$



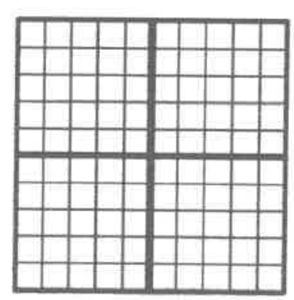
104. $y = -\frac{3}{2}x - 2$



105. $y = 2x + 1$



106. $y = \frac{1}{4}x$



Solving Proportions

1. Set the two cross-products equal to each other
2. Solve the equation for the variable

ex: $\frac{m}{4} = \frac{3}{5}$

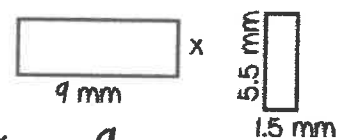
$\frac{5m}{5} = \frac{12}{5}$

$m = 2.4$

Similar Figures

1. To find a missing side length, set up a proportion, matching up corresponding sides.
2. Solve the proportion using the steps above.

ex:



$\frac{x}{1.5} = \frac{9}{5.5}$

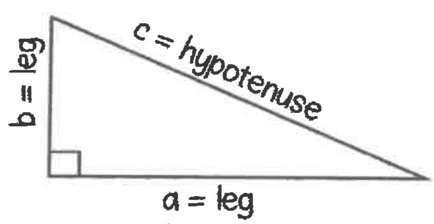
$x = 2.\overline{45}$ mm

The Pythagorean Theorem

*** The Pythagorean Theorem applies to right triangles only **

The sides next to the right angle (a & b) are legs

The side across from the right angle (c) is the hypotenuse

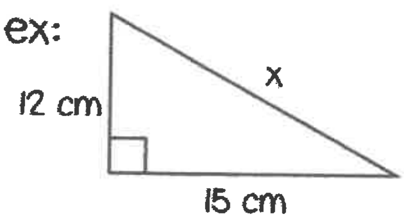


Pythagorean Theorem: $a^2 + b^2 = c^2$

To find the hypotenuse: add the squares of the legs and then find the square root of the sum

To find a leg: subtract the square of the given leg from the square of the hypotenuse and then find the square root of the difference

ex:



x is the hypotenuse

$12^2 + 15^2 = x^2$

$144 + 225 = x^2$

$369 = x^2$

$x = \sqrt{369}$ ≈ 19.2 cm

ex: $a = ?, b = 3, c = 6$

a is a leg

$a^2 + 3^2 = 6^2$

$a^2 + 9 = 36$

$a^2 = 36 - 9 = 27$

$a = \sqrt{27}$ ≈ 5.2

Solve each proportion, showing all work.

107. $\frac{6}{7} = \frac{4}{m}$

108. $\frac{12}{5} = \frac{k}{3}$

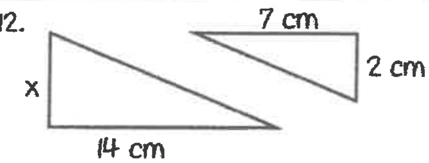
109. $\frac{h}{7} = \frac{8}{2}$

110. $\frac{22}{n} = \frac{9}{36}$

111. $\frac{4}{21} = \frac{3}{c}$

Assume each pair of figures is similar. Find the missing side length, showing all work.

112.



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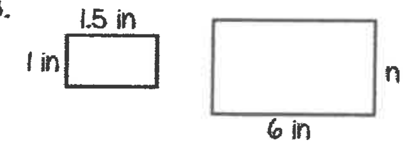

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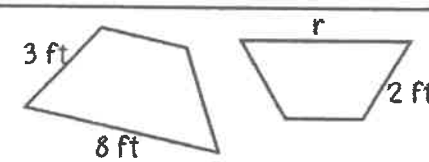

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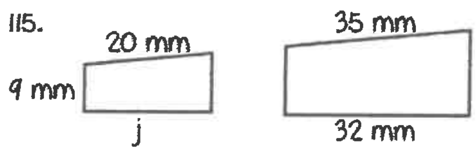

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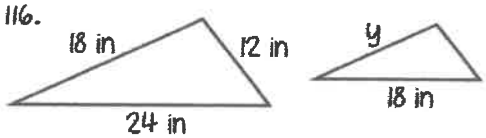

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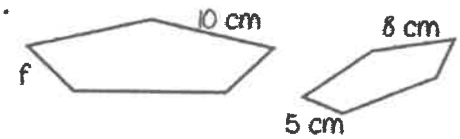


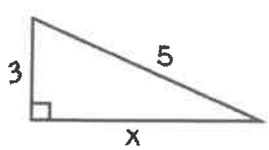
Find the missing side length in each right triangle to the nearest tenth. Show your work!

118 $a = 6, b = 8, c = ?$

119. $a = ?, b = 9$cm, $c = 13$cm

120. $a = 7, b = ?, c = 14$

121.



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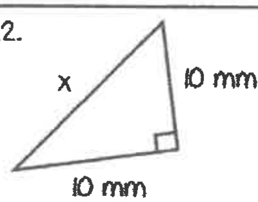

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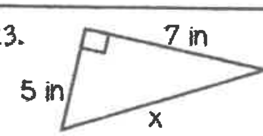


124. A 20-foot ladder is leaning against a wall. The base of the ladder is 5 feet from the wall. How high up the wall does the ladder reach?

Determine whether or not you can form a right triangle from the given side lengths. Explain.

125. 18, 22, 26

126. 5, 12, 13

Volume

Volume: the space inside a 3D figure (measured in cubic units)

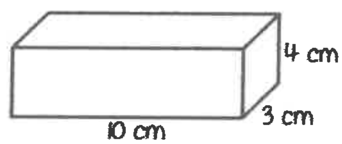
Abbreviations:

A = area, l = length, w= width, b = base, h = height, r = radius, V = volume, B = area of base

Formulas:

- Review of 2D Figure Area Formulas:
  - Rectangle: $A = lw$
  - Parallelogram: $A = bh$
  - Triangle: $A = \frac{1}{2}bh$
  - Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$
  - Circle: $A = \pi r^2$

- Prism: $V = Bh$



This base of this prism is a rectangle, so B = l·w

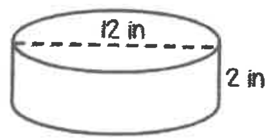
$V = Bh$

$V = lwh$

$V = 10 \cdot 3 \cdot 4$

$V = 120\ cm^3$

- Cylinder: $V = \pi r^2 h$



The diameter is 12 in, so the radius is 6 in

$V = \pi r^2 h$

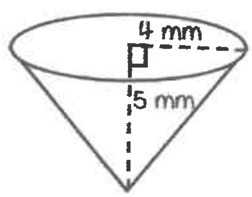
$V = \pi \cdot 6^2 \cdot 2$

$V = \pi \cdot 36 \cdot 2$

$V = 72\pi$

$V \approx 226.2\ in^3$

- Cone: $V = \frac{\pi r^2 h}{3}$



$V = \frac{\pi r^2 h}{3}$

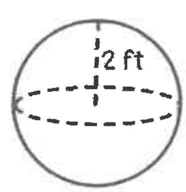
$V = \frac{\pi(4)^2 \cdot 5}{3}$

$V = \frac{\pi \cdot 16 \cdot 5}{3}$

$V = \frac{80\pi}{3}$

$V \approx 83.8\ mm^3$

- Sphere: $V = \frac{4\pi r^3}{3}$



$V = \frac{4\pi r^3}{3}$

$V = \frac{4\pi(2)^3}{3}$

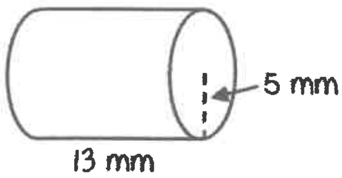
$V = \frac{4\pi \cdot 8}{3}$

$V = \frac{32\pi}{3}$

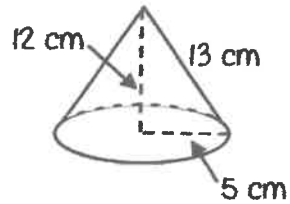
$V \approx 33.5\ ft^3$

Find the volume. If necessary, round to the nearest tenth.

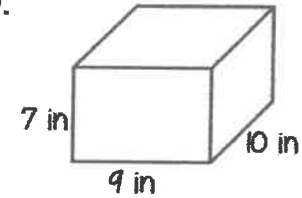
127.



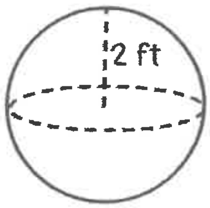
128.



129.



130.



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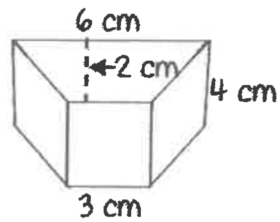

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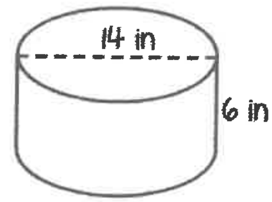

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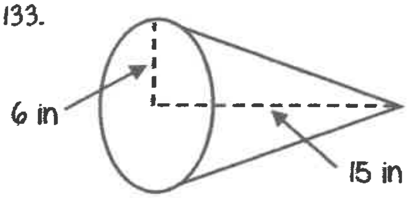
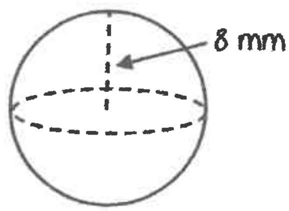
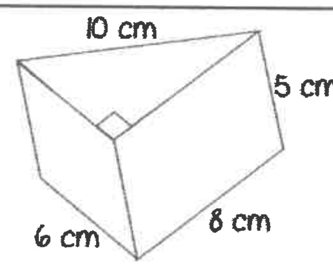


134.



135.



Solve each word problem. Round to the nearest tenth, if necessary.

136. A cylindrical cup is filled half-way with water. The cup has a diameter of 7 cm and a height of 15 cm. How many mL of water are in the cup? (1 mL = 1 cm^3)

137. A ball with a diameter of 7 cm is placed in a cube-shaped box with 8 cm sides. How much empty space is left in the box?